Inverse Heat Conduction with Uncertainty Analysis

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A technique is presented for the uncertainty analysis of the linear Inverse Heat Conduction Problem (IHCP) of estimating heat flux from interior temperature measurements. The selected IHCP algorithm is described. The uncertainty in thermal properties is considered as well as the temperature measurements. A propagation of variance equation is used for the uncertainty analysis. An example calculation is presented. Parameter importance factors are computed for the example problem; the volumetric heat capacity is the dominant parameter and an explanation is offered. Thoughts are presented on extending the analysis to include the nonlinear problem of temperature dependent properties.

In order to demonstrate the uncertainty analysis procedure for the IHCP, one must first select the algorithm that will be used. We selected the Duhamel's theorem based function specification method presented on page 128 of Beck, Blackwell and St. Clair Jr. [1].

The selected IHCP algorithm requires sensitivity coefficients, $\partial T/\partial q$, where T is temperature and q is heat flux. These sensitivity coefficients were computed from the analytical solution for a 1-D planar slab with a constant heat flux boundary condition on the front face and adiabatic back face.

The uncertainty propagation equation used is the same as used in the experimental community for the propagation of variance; see Beck and Arnold [2] or Coleman and Steele [3] for a derivation. If one considers uncertainty in thermal conductivity (k), volumetric heat capacity (ρc) and temperature measurements (Y_i), the following sensitivity coefficients are required: $\partial q/\partial k$, $\partial q/\partial \rho c$ and $\partial q/\partial Y_i$. The first two sensitivity coefficients were computed using a central finite difference.

The traditional form of the IHCP algorithm we used is not convenient for computing the sensitivity of the estimated heat flux to the temperature measurements. However, the filter coefficient form of the IHCP as discussed in Beck, Blackwell and St. Clair Jr. [1] is well suited for this sensitivity coefficient.

The propagation of variance equation leads to a natural definition of importance factors for the various uncertain parameters. This definition is the relative contribution of each parameter to the total variance.

An algorithm is presented for computing the filter coefficients.

Example uncertainty calculations are presented for a planar stainless steel slab in which the temperature rise is 1400 K over 2.5 s and are given in Figure 1. For this example, the volumetric heat capacity was the dominant parameter.

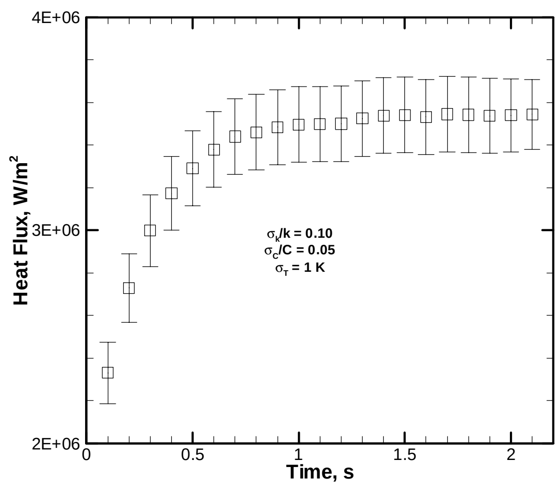

Figure 1. Estimated heat flux using temperature measurements with (simulated) errors.

References

[1] J. V. Beck, B. F. Blackwell and C. R. St. Clair Jr, *Inverse Heat Conduction*, Wiley, New York, 1985.

[2] J. V. Beck and K. J. Arnold, *Parameter Estimation in Engineering and Science*, Wiley, New York, 1977.

[3] H. W. Coleman and W. G. Steele, *Experimentation and Uncertainty Analysis for Engineers*, 2^{nd} Ed., Wiley, 1999.